Aerial Photography and Image Interpretation for Resource Management

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6. Calculate the percent error caused by using the short-cut height equation.
7. Show on a sketch the absolute parallax of a point and difference in absolute parallax between two points on a stereoscopic pair of overlapping photographs.
8. State three direct and one indirect inference that can be derived from the height equations.
9. Explain with the use of a diagram how all the terms in the height equation are measured.
10. Draw a diagram of the parallax bar and parallax wedge, and describe how each is used to measure dP.
11. Given adequate information, either on aerial photographs or diagrams, measure the difference in parallax using an engineer's scale, a parallax bar, and a parallax wedge (only those with good stereoscopic vision can properly make use of the parallax bar or wedge).
12. Given a sterescopic pair of photos, the focal length of the camera lens, and the height of an object on the photos, determine the flying height of the aircraft and the photo scale at the base of the object assuming a level terrain situation.

MEASURING HEIGHTS ON SINGLE AERIAL PHOTOS

There are two basic methods of measuring heights on *single* aerial photographs — the topographic displacement method and the shadow method. The shadow method has two variations.

The Topographic-Displacement Method

In Chapter Two we introduced the formula, $h = \frac{d(H)}{r}$, which in isolated cases can be used to measure the height of an object on a single large-scale aerial photo. This method requires that (1) the object being measured must be vertical from bottom to top, like a tower or building, which rules out measuring differences in elevation between two points on the ground that are not directly over one another; (2) the distance from the nadir must be great enough to create enough topographic displacement to be measured; (3) the photo scale must be large enough so that the displacement on the photograph can be measured; and (4) both the top and bottom of the object being measured must be visible on the same photo.

Because these criteria are seldom met, the method is used only on rare occasions when interpreting aerial photos for natural resources. If you have a situation when this method can be used, you should refer to Chapter Two for the technique.

The Shadow Methods

Sun-Angle Shadow Method

If you can measure the length of a shadow and know the angle of the sun that creates the shadow, the height of the object can be calculated with simple trigonometry. However, this method assumes that the ground on which the shadow falls is level and that the object being measured is perfectly vertical. Snow or brush on the ground will also shorten the shadow. Finally, the object must have a sharp-pointed distinct top so that the top of the object creates a definite shadow point on the ground. Figure 7.1 illustrates these sources of error.

Another difficulty with this method is that the calculation of the sun angle at the instant of exposure is a long process and the angle changes with the position on the earth, time of day, season of year, and even changes from year to year. However, the method is sometimes used. This is one of the reasons for printing the date and time of day on the top of some photographs.

Because of these complications and because the parallax method of height measurement on stereoscopic pairs is usually better we will not discuss this method further or illustrate the calculating procedure. However, there is another shadow method that is simple and worth a brief discussion. We will call it the proportional shadow-length method.

Proportional Shadow-Length Method

In this method you must know the height of at least one object on the photo that has a measurable shadow. Suppose the known height of an object is 160 ft and it casts a shadow on the photo of 0.16 in. What would be the height of an object with a shadow length of 0.11 in? By simple proportion it would be 0.11/0.16 of 160 ft, or 110 ft tall. This method assumes vertical objects but the ground does not have to be level as long as these conditions are the same for both the known and unknown objects. Snow or brush on the ground will also create errors when using this method of height measurement.

MEASURING HEIGHTS BY PARALLAX DIFFERENCES

This is the most useful method of measuring heights on aerial photographs. It requires a stereoscopic pair of high-quality aerial photographs and good depth perception on the part of the interpreter.

The Parallax-Height Equations

There are many forms of the parallax height equation. They all assume perfectly vertical photography, but if tilt is 3° or less the resulting error in height

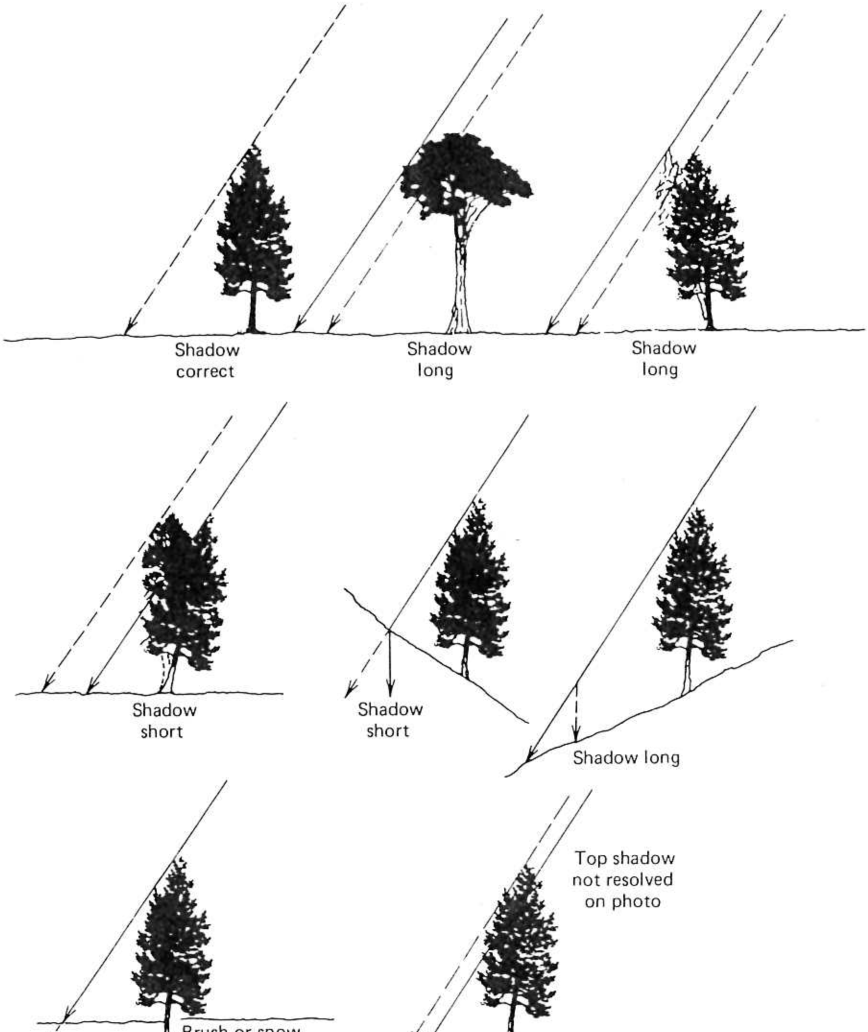


Figure 7.1. Causes of incorrect shadow length and therefore, incorrect heights when using the sun-angle shadow method. Correct shadow lengths indicated by dashed lines.

measurement is small. Three different parallax-height equations with additional modifications are given: (1) the mountainous-terrain equation, (2) the level-terrain equation, and (3) the short-cut equation.

The mountainous-terrain height equation is valid for both mountainous and level terrain but is more complicated than the other equations. The level-terrain equation is valid only when the base of the object being measured is at the same elevation as the average elevation of the two principal points of the stereopair of photos. The short-cut equation with a slight modification can approximate either the level terrain or the mountainous-terrain equations but is never completely valid. It is the simplest of the height equations and can be used with previously prepared tables.

The Mountainous-Terrain Equation

Two different forms of this equation are as follows:

$$h = \frac{(H)dP}{P_b + dP} = \frac{(H)dP}{P + \left[\frac{P(\pm \triangle E)}{H}\right] + dP}$$

Where

h = height of the object being measured

H = flying height above the *base* of the object

dP = difference in absolute parallax between the top and bottom of the object

P = the average absolute parallax of the two ends of the baseline (measured as the average distance between the principal and conjugate principal points of the stereopair of photos)

P_b = the absolute parallax at the base of the object (measured as the distance between the principal points of the two photos minus the distance between the images of the base of the object on the two photos when they are properly aligned for the measurement of dP)

$\pm\triangle E$ = difference in elevation between the base of the object and the average of the two principal points, plus if higher and minus if lower (Figure 7.2)

For these two equations to be equal, P_b *must equal* $P + \left[\frac{P(\pm\triangle E)}{H}\right]$. The term P is the average absolute parallax of the baseline and $\frac{P(\pm\triangle E)}{H}$ is the difference in absolute parallax between the baseline and the base of the object being measured. Therefore, when added together the result is the absolute parallax at the base of the object, or, by definition, P_b.

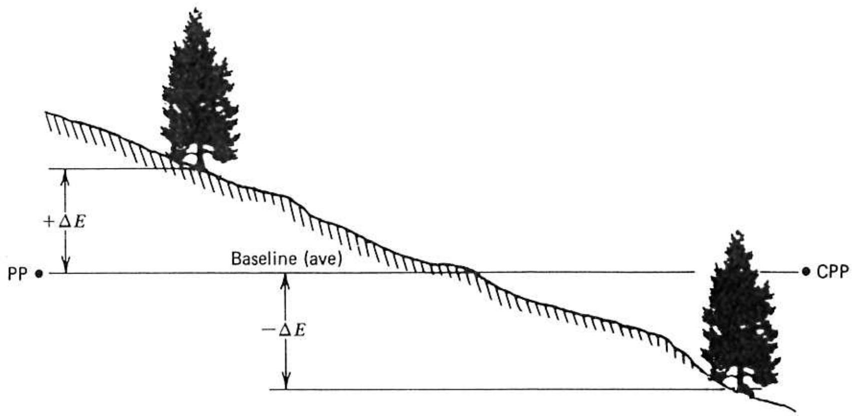


Figure 7.2. Illustration of $\pm\Delta E$. See text. (From D. P. Paine, 1979, *An Introduction to Aerial Photography for Natural Resource Managment,* O.S.U. Bookstores, Inc., copyright 1973, reproduced with permission.)

The reason for presenting two mountainous-terrain height equations is that in certain situations one equation may be more applicable than the other. An accurate measurement of P_b is difficult if the base of the object is not clearly visible on both photos as is the case in densely vegetated areas. On the other hand, $\pm\Delta E$ is frequently unknown and not easily determined. In the situation where both $\pm\Delta E$ and P_b are difficult to obtain it is still better to make the best estimate possible of either value rather than to use the level terrain equation when the mountainous equation should be used.

The Level-Terrain Equation

This equation is valid only when $\pm\Delta E$ is zero. When $\pm\Delta E$ is zero, the mountainous-height equation reduces to the level terrain equation or:

$$h = \frac{(H)dP}{P + \mathrm{dP}}$$

Examples of the use of this, and the other height equations are given later.

Because *small* differences in elevation between the base of the object being measured and the average elevation of the baseline ($\pm\Delta\ E$) result in small errors in h, the following rule of thumb is generally recognized. *The level-terrain height equation can be used when the difference in elevation between the base of the object being measured and the average elevation of two principal points is less than 5 percent of the base of the object.* Under this condition the error in the calculated height is usually less than the error created by the inability of the average photo interpreter to precisely measure dP. It is *never* incorrect, however, to use the mountainous-terrain-height equation. Later we will show examples to illustrate the magnitude of errors in height caused by using the level terrain equation when the mountainous-terrain equation should have been used.

The Short-Cut Height Equation. This equation can also be written two ways depending on whether it approximates the mountainous or the level-terrain equation as follows:

$$h = \frac{(H)dP}{P_b} = \frac{(H)dP}{P}$$

Neither of these equations is completely accurate because they ignore dP in the denominator, but the error is small if dP is a small proportion of P or P_b.

The advantage of the short-cut equation is that tables of differential parallax factors (DPF) have been developed for different baselines (P or P_b) and different camera lens focal lengths to simplify the calculation of object heights (Tables 7.1 and 7.2).

To use the DPF tables we convert the short-cut equation to a simpler form. Starting with the short-cut, mountainous-terrain equation,

$$h = \frac{(H)dP}{P_b}$$

and substituting f(PSR) for H we get:

$$h = \frac{f(\text{PSR})dP}{P_b}$$

Next, we substitute DPF for $\frac{f}{P_b}$, because it remains constant for a given stereoscopic pair of aerial photos. We get:

$$h = (\text{DPF})\ (\text{PSR})\ (dP)$$

Starting with the level-terrain height equation gives us the same answer if we substitute DPF for $\frac{f}{P}$. Thus, when using the DPF tables we approximate either the mountainous terrain or the level-terrain equation depending on whether we use P or P_b.

Let's compare the appropriate short-cut equation with the mountainous-terrain equation and see how much error is involved. Suppose we measure a dP of 1.14 mm on photos with a PSR of 12,640 at the base of an object, that an 8¼-in. camera lens was used and that P is 63.50 mm. Using table 7.2 we find the DPF to be 0.0108. Solving our equation we get:

$$\begin{aligned} h &= (\text{DPF})\ (\text{PSR})\ (dP) \\ &= (0.0108)\ (12{,}460)\ (1.14) \\ &= 153.4 \text{ ft } (46.8 \text{ m}) \end{aligned}$$

TABLE 7.1 Differential Parallax Factors for Height Measurement in Feet When dP is Measured in Inches. For Heights in Meters, Multiply by 0.3048.

P or P_b in		Focal Length of Photography		
in.	mm	6 in. (152 mm)	8¼ in. (209 mm)	12 in. (305 mm)
2.0	50.80	0.250	0.344	0.500
2.1	53.34	0.238	0.327	0.476
2.2	55.88	0.227	0.312	0.454
2.3	58.42	0.217	0.299	0.435
2.4	60.96	0.208	0.286	0.416
2.5	63.56	0.200	0.275	0.400
2.6	66.04	0.192	0.264	0.384
2.7	68.58	0.185	0.255	0.370
2.8	71.12	0.179	0.245	0.358
2.9	73.66	0.172	0.237	0.345
3.0	76.20	0.167	0.229	0.333
3.1	78.74	0.161	0.222	0.322
3.2	81.28	0.156	0.215	0.312
3.3	83.82	0.151	0.208	0.303
3.4	86.36	0.147	0.202	0.294
3.5	88.90	0.143	0.196	0.286
3.6	91.41	0.139	0.191	0.278
3.7	93.98	0.135	0.186	0.270
3.8	96.52	0.132	0.181	0.263
3.9	99.06	0.128	0.176	0.256
4.0	101.60	0.125	0.172	0.250
4.1	104.14	0.122	0.168	0.244
4.2	106.68	0.119	0.164	0.238
4.3	109.22	0.116	0.160	0.233
4.4	111.76	0.114	0.156	0.227

Using the mountainous equation we get:

$$h = \frac{0.6875 \text{ ft } (12{,}460)\ (1.14 \text{ mm})}{63.50 \text{ mm} + 1.14 \text{ mm}} = 151.1 \text{ ft } (46.0 \text{ m})$$

Thus using the short-cut equation resulted in an error of 2.3 ft (0.70 m) or only 1.5 percent for this particular example. Notice that in this solution we substituted f(PSR) for H.

Units of Measure

When using any of the parallax height equations, both P and dP must have the same units of measurement, usually inches or millimeters. All vertical distances (H,

TABLE 7.2 Differential Parallax Factors for Height Measurement in Feet When dP is Measured in Millimeters. For Heights in Meters, Multiply by 0.3048.

P or P_b in		Focal Length of Photography		
in.	mm	6 in. (152 mm)	8¼ in. (209 mm)	12 in. (305 mm)
2.0	50.80	0.0098	0.0135	0.0197
2.1	53.34	0.0094	0.0129	0.0187
2.2	55.88	0.0089	0.0123	0.0179
2.3	58.42	0.0085	0.0118	0.0171
2.4	60.96	0.0082	0.0113	0.0164
2.5	63.50	0.0079	0.0108	0.0157
2.6	66.04	0.0076	0.0104	0.0151
2.7	68.58	0.0073	0.0100	0.0146
2.8	71.12	0.0070	0.0097	0.0141
2.9	73.66	0.0068	0.0093	0.0136
3.0	76.20	0.0066	0.0090	0.0131
3.1	78.74	0.0063	0.0087	0.0127
3.2	81.28	0.0061	0.0085	0.0123
3.3	83.82	0.0059	0.0082	0.0119
3.4	86.36	0.0058	0.0080	0.0116
3.5	88.90	0.0056	0.0077	0.0113
3.6	91.41	0.0055	0.0075	0.0109
3.7	93.98	0.0053	0.0073	0.0106
3.8	96.52	0.0052	0.0071	0.0104
3.9	99.06	0.0050	0.0069	0.0101
4.0	101.60	0.0049	0.0068	0.0098
4.1	104.14	0.0048	0.0066	0.0096
4.2	106.68	0.0047	0.0065	0.0094
4.3	109.22	0.0046	0.0063	0.0092
4.4	111.76	0.0045	0.0061	0.0089

$\pm\Delta E$, and h) must also be in the same units, usually feet or meters. Thus, the units for dP in the numerator cancel the units for $P + dP$ in the denominator, leaving the answer (h) in the same units as (H). Actually, we can mix units by measuring P and dP in millimeters and H in feet and get feet as we did in the last example.

Inferences Derived From the Parallax-Height Equation

Even though the inferences to follow apply to all of the parallax-height equations, we will use the level-terrain equation to illustrate. In this equation, $P + dP$ is really the absolute parallax at the top of the object. If we call this P_t, the equation becomes:

$$h = \frac{(H)dP}{P_t} = \frac{f(\mathrm{PSR})dP}{P_t}$$

Solving for dP we get:

$$dP = \frac{h(P_t)}{H} = \frac{h(P_t)}{f(\text{PSR})}$$

from which the following direct and indirect inferences can be made:

1. dP is directly proportional to the height of the object.
2. dP is directly proportional to P_t and therefore to P, which is inversely proportional to the percent endlap.
3. dP is inversely proportional to the flying height and therefore to f and PSR.
4. dP is independent of the distance of the object from the nadir. This is an indirect inference and was not the case with topographic displacement measured on single photographs.

Derivation of the Parallax-Height Equation

For our example we will use the level-terrain equation from which all other height equations can be obtained. The geometry of the level-terrain equation is illustrated in Figure 7.3. All the symbols have been previously defined, but let's elaborate on dP, the difference in absolute parallax. It is defined as the difference in absolute parallax between the top and bottom of an object. This also applies to the difference in absolute parallax between any two points in our stereoscopic model. All points of equal ground elevation have the same absolute parallax in a vertical stereoscopic pair of photographs. Similarly, points at different elevations have different absolute parallaxes. Therefore, the accurate measure of dP is the key to successful height measurement on aerial photographs.

Considering the geometry of similar triangles ABC and ADE in Figure 7.3 we can state that:

$$\frac{h}{(H - h)} = \frac{dP}{P}$$

and with a little algebraic manipulation we have the parallax-height equation for level terrain.

$$P(h) = dP(H - h)$$

$$P(h) = dP(H) - dP(h)$$

$$P(h) + dP(h) = dP(H)$$

$$h\,(P + dP) = dP(H)$$

$$h = \frac{(H)\,dP}{P + dP}$$

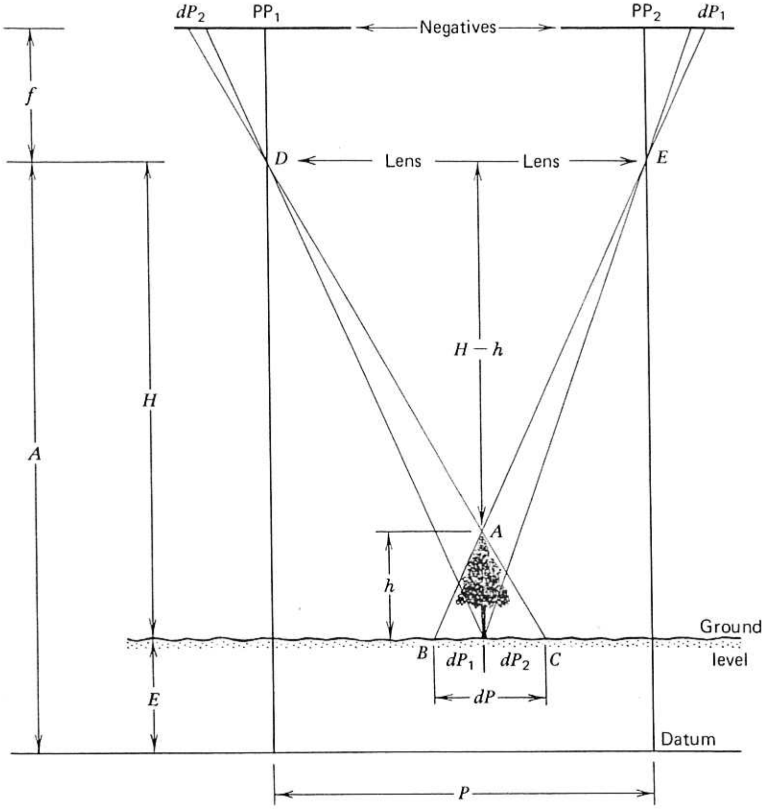


Figure 7.3. Geometry for the derivation of the level terrain parallax height equation. (Courtesy of U.S. F.S. Pacific Northwest Forest and Range Experiment Station.)

The determination of H was discussed in Chapter Four on scale, where H equals f(PSR).

The variable, P, in our equation is really the average absolute parallax of our two principal points. The absolute parallax of the PP on the left photo of any stereoscopic pair is the distance between the PP and CPP on the right photo and the absolute parallax of the PP on the right photo is the distance between the PP and CPP on the left photo. On the ground these distances are the same. On the photos they are the same only if the aircraft maintained the same flying height (A) over both exposure stations, there is no y tilt in the photos, and the principal points of both photos are at the same elevation. Because these conditions are seldom met, we use an average distance measured on the photos when using the level terrain equation.

When using the mountainous terrain equation we use P_b (previously defined) or add the correction factor to P as we discussed earlier.

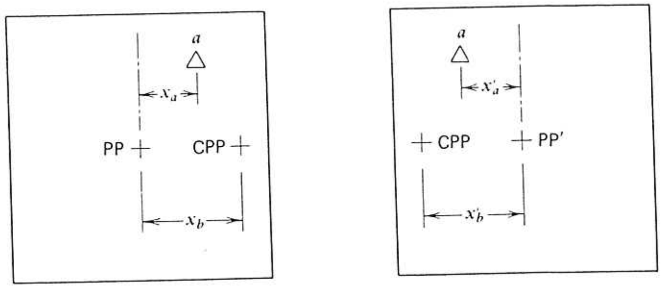


Figure 7.4. Absolute parallax of point a and of the baseline, x_b or x_b'. (From D. P. Paine, 1979, *An Introduction to Aerial Photography for Natural Resource Management,* O.S.U. Bookstores, Inc., copyright 1973, reproduced with permission.)

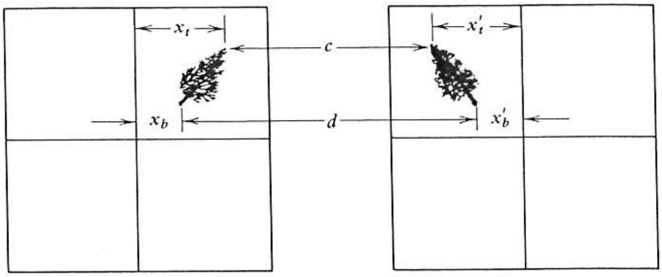
This leaves only the variable dP for discussion. We have previously defined it but now let's take a closer look at the theory of dP. The instruments used to measure it will be discussed later.

In Figure 7.4, a diagram of two overlapping photos, the x axis is positive (to the right) of the principal point on the left photo and negative (to the left) on the right photo. The absolute parallax of object a[1] is $x_a - (-x'_a)$, or $x_a + x'_a$. The average absolute parallax of the baseline is $x_b + x'_b$ divided by 2. If point a is at the same elevation as the baseline, then $x_a + x'_a - (x_b + x'_b)/2 = dP = 0$. To have a difference in absolute parallax we need different elevations, such as the top and bottom of a tree. Then the difference in absolute parallax of the tree is the absolute parallax of the top minus the absolute parallax of the bottom.

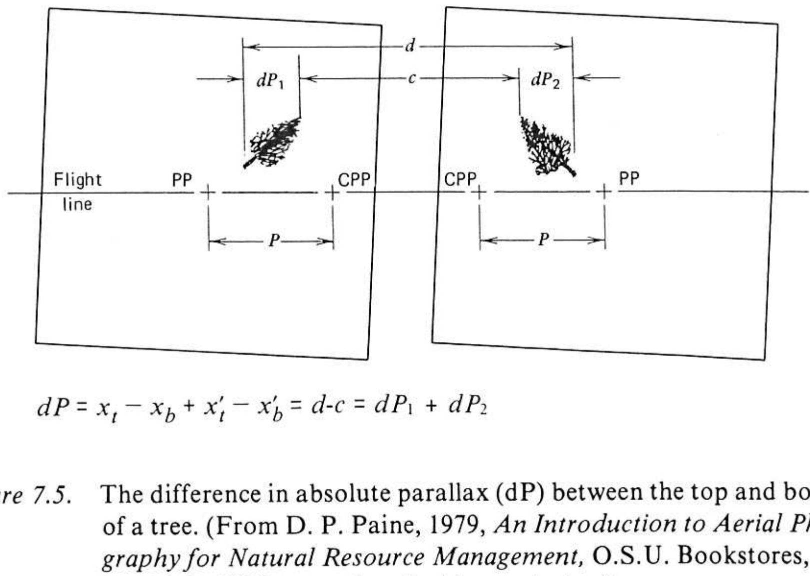
Let's consider a single tree illustrated on diagrams of two adjacent overlapping vertical photos (Figure 7.5.). On the left photo (top illustration) the x coordinate of the top of the tree is x_t and the x coordinate of the bottom of the tree is $+x_b$. On the right photo the x coordinate of the top of the tree is $-x'_t$ and the x coordinate of the bottom is $-x'_b$. Considering both photos, with the PPs and CPPs perfectly aligned the absolute parallax of the top of the tree is $x_t - (-x'_t)$, or $x_t + x'_t$, and the absolute parallax of the bottom of the tree is $x_b + x'_b$. Then the difference in absolute parallax, the term dP in our equation, is $(x_t + x'_t) - (x_b + x'_b)$.

However, instead of measuring x_t, x'_t, x_b, and x'_b, we can determine dP by measuring only c and d (Figure 7.5) and taking their difference. We can also rearrange our equation for dP so that dP equals $x_t - x_b + x'_t - x'_b$, or $dP_1 + dP_2$, which is exactly the same as d-c. This is also the same as $dP_1 + dP_2$ shown in Figure 7.3 used to derive the parallax-height equation. The way we actually measure dP on the photographs is to subtract c from d.

[1] *The elevations of object a, the PP,* and the CPP are all the same.



Absolute parallax of top = $x_t - (-x'_t) = x_t + x'_t$
Absolute parallax of bottom = $x_b - (-x'_b) = x_b + x'_b$
Difference in absolute parallax, $dP = (x_t + x'_t) - (x_b + x'_b)$



$$dP = x_t - x_b + x'_t - x'_b = d\text{-}c = dP_1 + dP_2$$

Figure 7.5. The difference in absolute parallax (dP) between the top and bottom of a tree. (From D. P. Paine, 1979, *An Introduction to Aerial Photography for Natural Resource Management,* O.S.U. Bookstores, Inc., copyright 1973, reproduced with permission.)

Numerical Examples

Before discussing various techniques and special instruments for measuring dP on the photographs let's look at some example problems. The first example will illustrate the effect of using the level-terrain equation in a situation where the mountainous-terrain equation should have been used.

Example 1

Assume that we have three trees, all the same height—one tree is at the same elevation as the photo baseline and the other two are 1000 ft above and below the baseline elevation (Figure 7.6 top). Because these trees are the same height, the photo-measured *dPs* would have to be different because they would be viewed at different angles by the camera (Figure 7.6, bottom). This substantiates the inference made earlier, that "*dP* is inversely proportional to the flying height" (H) above the base of the object. Let us also assume that:

Elevation of the baseline		2000 ft
Average PSR of the baseline		12,000
Average photo length of the baseline (P)		3.00 in.
Focal length of camera lens used (f)		6 in.
Difference in absolute parallax (dP)		
Case 1 dP	0.100 in.	
Case 2 dP	0.145 in.	
Case 3 dP	0.073 in.	

How tall are these trees? First, we must calculate the flying height above the base of the tree.

$$H = f(\text{PSR}) = 0.5 \text{ ft. } (12{,}000) = 6000 \text{ ft}$$

Case 1

(P and baseline are at the same elevation.)

$$h = \frac{(H)\ (dP)}{P + dP} = \frac{(6000 \text{ ft})\ (0.100 \text{ in.})}{3.00 \text{ in.} + 0.100 \text{ in.}} = 194 \text{ ft}$$

Notice that in case 1 we used the level-terrain equation because the base of the object and the photo baseline were at the same elevation. However in both case 2 and 3 we must use the mountainous terrain equation because:

$$\frac{+\Delta E}{H}(100) = \frac{+1000 \text{ ft}}{6000 \text{ ft} - 1000 \text{ ft}}(100) = 20\% \text{ for case 2}$$

and

$$\frac{-1000 \text{ ft}}{6000 \text{ ft} + 1000 \text{ ft}}(100) = 14.3\% \text{ for case 3}$$

which is greater than the 5 percent that we allow before shifting to the mountainous-terrain equation.

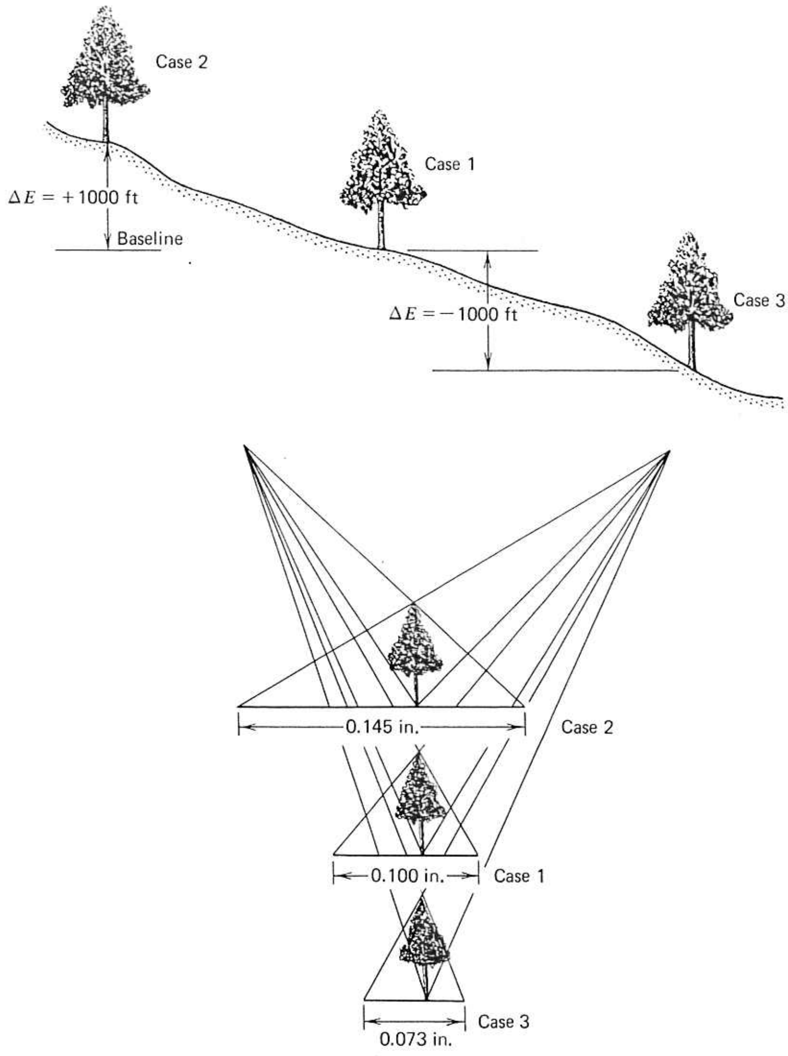


Figure 7.6. Illustration to go with Example 1. See text. (From D. P. Paine, 1979, *An Introduction to Aerial Photography for Natural Resource Management,* O.S.U. Bookstores, Inc., copyright 1973, reproduced with permission.)

Case 2

(The base of the object is 1000 ft *above* the baseline P for an H of 5000 ft.)

$$h = \frac{(H)\,(dP)}{P + \left[\dfrac{P(+\Delta E)}{H}\right] + dP}$$

$$= \frac{(5000 \text{ ft})\,(0.145 \text{ in.})}{3.00 \text{ in.} + \left[\dfrac{1000 \text{ ft }(3.00 \text{ in.})}{5000 \text{ ft}}\right] + 0.145 \text{ in.}}$$

$$= \frac{(5000 \text{ ft})\,(0.145 \text{ in.})}{3.00 \text{ in.} + 0.60 \text{ in.} + 0.145 \text{ in.}} = \frac{725 \text{ ft}}{3.745} = 194 \text{ ft}$$

Using the level-terrain equation in case 2 would have given us an erroneous height of 231 ft, an error of +37 ft or 19 percent.

Case 3

(The base of the object is 1000 ft *below* the baseline P for an H of 7000 ft.)

$$h = \frac{(7000 \text{ ft})\,(0.073 \text{ in.})}{3.00 \text{ in.} + \left[\dfrac{-1000 \text{ ft }(3.00 \text{ in.})}{5000 \text{ ft}}\right] + 0.073 \text{ in.}}$$

$$= \frac{7000 \text{ ft }(0.073 \text{ in.})}{3.00 \text{ in.} - 0.429 \text{ in.} + 0.073 \text{ in.}} = \frac{511 \text{ ft}}{2.644} \approx 194 \text{ ft}$$

Using the level terrain equation in case 3 would have given us an erroneous height of 166 ft, an error of -28 ft or 14 percent.

These examples illustrate extreme cases where $\pm\Delta E$ is large to emphasize the effect of using the level-terrain equation when $\pm\Delta E$ greatly exceeds 5 percent of H. How much error is involved when $\pm\Delta E$ is about 5 percent? Suppose ΔE is +300 ft (+5.3 percent of H) in case 2. The dP would be 0.111 in. Using the level-terrain equation results in a calculated height of 203 ft, an error of +9 ft or 4.6 percent. The percent error would be even less if E was a −300 ft (4.8 percent of H) in case 3. Errors of this magnitude are not serious because errors in the measurement of dP frequently create errors greater than this. Therefore, our 5 percent rule of thumb appears reasonable for the average photo interpreter.

Example 2

Suppose we have the Washington Monument on a stereoscopic pair of aerial photographs. We do not know the scale or the flying height, but we do know that the monument is 555 ft tall and that an 8¼-in. lens camera was used. How could we calculate the flying height and the photo scale?

First, we properly align the photos and find by careful measurement that P is 3.36 in. and dP is 0.340 in. Then we solve the level-terrain equation for H and get:

$$H = \frac{h(P + dP)}{dP} = \frac{555 \text{ ft } (3.36 \text{ in.} + 0.340 \text{ in.})}{0.340 \text{ in.}} = 6040 \text{ ft}$$

and the PSR at the base of the monument is:

$$\text{PSR} = \frac{H}{f} = \frac{6040 \text{ ft}}{0.6875 \text{ ft}} = 8785$$

This is not the most accurate way to determine H or PSR; it is an approximate method that can be used when other methods are not feasible.

Techniques and Instruments for Measuring dP

As stated earlier, the key to accurate height measurements on stereoscopic pairs of photographs is the accurate measurement of dP. The most accurate instruments are expensive stereoplotting instruments designed for making topographic and planimetric maps that require highly skilled operators. These instruments also correct for tilt, an additional source of error when measuring heights on nonrectified photographs.

The average photointerpreter must settle for something a little simpler. The simplest instrument is an engineer's scale but its use is limited to situations where the top and botom of the object are visible on both photos, and measurements can only be approximate to the nearest 0.01 of an inch, which is not as precise as frequently required. Parallax bars and parallax wedges (or ladders) can theoretically be used to measure dP to the nearest 0.001 in. or 0.01 mm and have a much wider application.

The Engineer's Scale

As explained in Figure 7.5, dP can be measured by subtracting the distance between the tops of two images of a single object from the distance between the bottoms of the object on a properly oriented stereoscopic pair of photographs. If the images of objects were always as well defined and exaggerated as the buildings in Figure 7.7, we would have fair success with the engineer's scale. However, this is seldom the case when interpreting photos for natural resources and because we should measure more precisely than to the nearest 0.01 in., we should learn to use either the parallax bar or the parallax wedge.

Parallax Bar

The parallax bar consists of two transparent cursors, each with a small etched dot, separated by a bar. One of these cursors is rigidly fastened to the bar. The other is moved along the bar, toward or away from the first cursor, by means of a micrometer screw that records the distance between the two dots to the nearest thousandth of an inch or hundredth of a millimeter. Parallax bars have been designed for use with both lens and mirror stereoscopes (Figures 7.8, and 7.9).

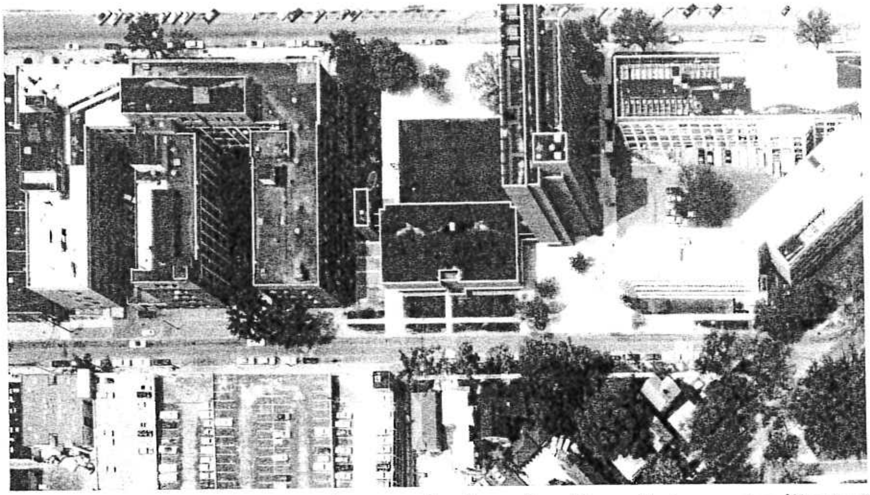


Figure 7.7. Tall buildings on large-scale, low-elevation photography illustrate large absolute parallaxes. Differences in absolute parallax can be measured with an engineer's scale on stereograms of similar photography.

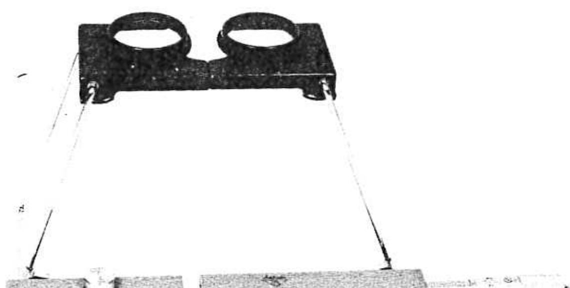


Figure 7.8. Lens stereoscope with parallax bar attached. (Courtesy Alan Gordon Enterprises, Inc., North Hollywood, California.)

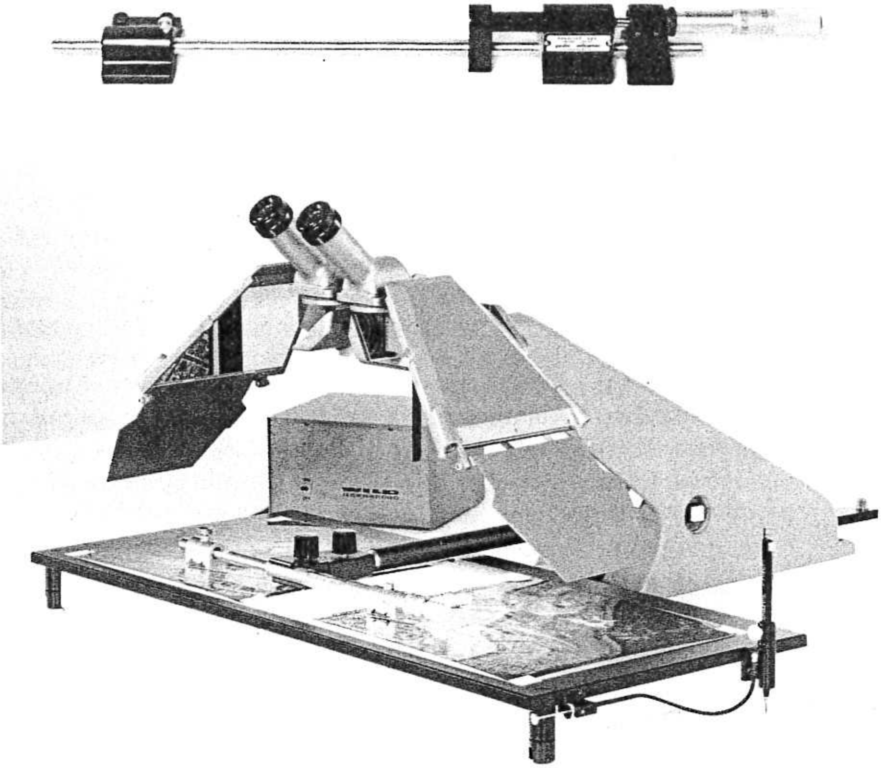


Figure 7.9. Parallax bar for use with a mirror stereoscope. (Courtesy Alan Gordon Enterprises, Inc., top.) Mirror stereoscope with parallax bar and cantilever stand — bottom. (Courtesy Wild Heerbrugg Instruments, Inc.)

To operate a parallax bar we first properly align a stereoscopic pair of photographs with respect to the flight line. The photos are separated by a distance compatible with the particular stereoscope and parallax bar you are going to use. Both photos should be securely fastened to the table top to prevent slippage during measurement. Then place the fixed dot (usually the left one) of the parallax bar beside one of the image tops, a tree for example, and adjust the micrometer so that the movable dot is beside the image top on the other photo (do not use the stereoscope). You can then read the micrometer setting, which we can think of as the distance between the two image tops (Figure 7.10). Next we do the same thing for

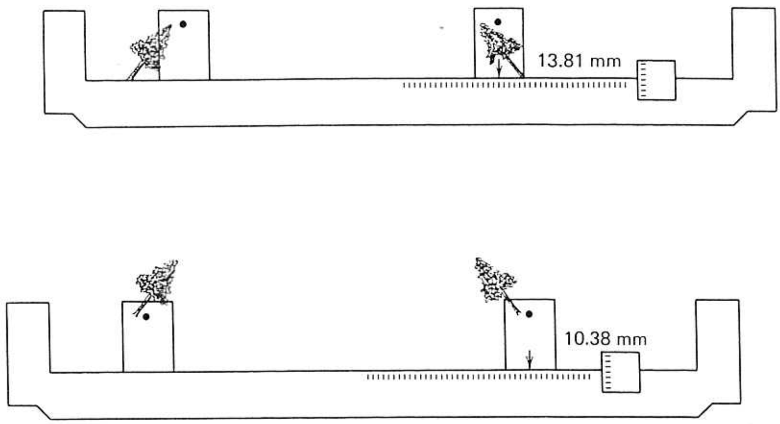


Figure 7.10. Measuring the difference in absolute parallax (dP) of a tree using a parallax bar. In this illustration the dP is 3.43 mm. The scale is not reversed here. (Adapted from D. P. Paine, 1979, *An Introduction to Aerial Photography for Natural Resource Management,* O.S.U. Bookstores, Inc., copyright 1973, reproduced with permission.)

the bottom of the tree and read this distance on the micrometer. The difference between these two readings gives us dP.

This explanation of the use of the parallax bar shows what we are measuring, not how dP measurements are made. In actual practice we use the stereoscope and the floating dot principle as was discussed in Chapter Six.

With the parallax bar oriented as in Figure 7.10, we look at both photo images through the stereoscope. The two tree images fuse, the tree stands up in the vertical dimension, and the two dots fuse into one dot, which appears to float in space right at the tip of the tree. If we turn the micrometer knob back and forth we will see the dot rise and fall in space. You will notice that it does not follow a strictly vertical line, but slants a little, because one dot is fixed and the other moves. With this type of instrument we can determine the parallax differences we are after by merely "floating" the dot at the top and the bottom of the object and taking the difference between the two readings obtained. The procedure for elevation difference is the same as for height, except that the dot is floated at the ground level in two different locations.

You may notice a couple of odd characteristics of your parallax bar. First you are not reading the actual distance between images — it is an arbitrary reading. However, when you take the difference beween the two measurements, you get a true dP. Second, many parallax bars have the scales reversed so that the reading you get when the dot floats at the image top is greater than when it floats at the bottom, which is opposite the true situation. Manufacturers have done this because it might seem more logical to have a greater reading at the image top than at the bottom (Figure 7.10).

Parallax Wedge

The third method of measuring dP is with a parallax wedge. It is a simple, inexpensive device for measuring parallax difference. It can be thought of as a whole series of parallax bars, each set at a reading differing from the adjacent ones by a constant amount. The result is two rows of slightly converging circles, dots or lines, separated by about 2 to 2½ in., as you see in Figure 7.11. The distance between the two lines can be read from a scale alongside one line. When properly oriented on a pair of photos under the stereoscope, the two lines fuse together for a portion of their length. Becuse the lines are different distances apart at each end, the fused portion floats in space as a sloping line or series of dots. To read the parallax difference for an object like a tree, the wedge is moved about on the photos until the sloping line appears to intersect the ground at the base of the object. A reading is then taken alongside the line at that point (Figure 7.12, right) then the wedge is moved until the sloping line appears to cut across the tip of the tree (Figure 7.12, left). The scale along the line is again read at this point. The difference between the two readings is the parallax difference, or 0.134 in. in this example.

The wedge is neither more or less accurate than the bar. The choice is mostly one of personal preference. The wedge is inexpensive and has no mechanical parts

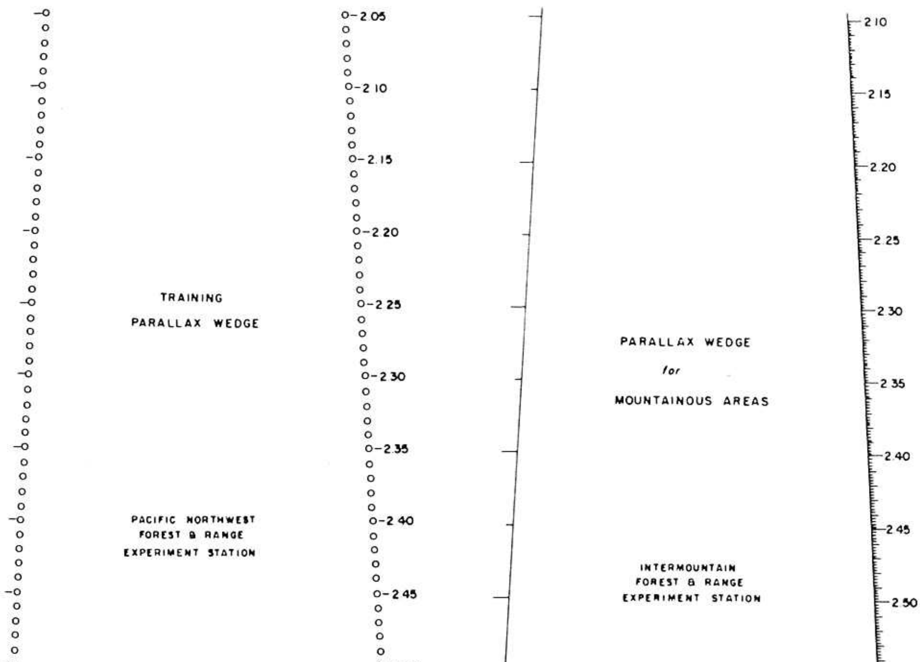


Figure 7.11. Training parallax wedge (left) and standard parallax wedge (right). The scale has been reduced in this illustration. (Courtesy U.S. Forest Service).

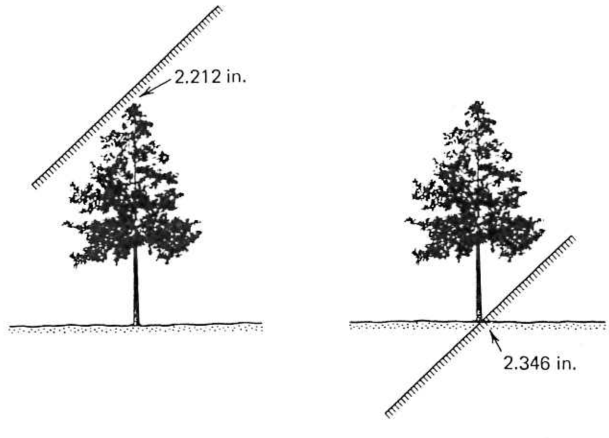


Figure 7.12. Parallax wedge orientation (side view as it would appear from above) for reading parallax differences between the top and bottom of a tree. The dP would be 0.134 in. (From D. P. Paine, 1979, *An Introduction to Aerial Photography for Natural Resource Management,* O.S.U. Bookstores, Inc., copyright 1973, reproduced with permission.)

to get out of adjustment. The bar is considerably more expensive but has certain advantages. Many people find it quicker to learn and easier to use in practice. With the bar, you can make any number of repeated measurements without the bias of knowing what the previous readings were. When using the wedge you can see the numbers on the scale when selecting top or bottom readings which might influence your selection, if you remember what you got the time before. To do so with the bar you have to take your eyes off the floating dot to read the vernier.

Measuring dP with Different Instruments

Now let's work some examples using each of these methods—the engineer's scale, the parallax bar, and the parallax wedge. To make it easy let's begin with the large-scale stereogram of model trees in Figure 7.13. We will assume that the letter G is on the ground at the same elevation as the base of the tree. The parallax of the baseline P is 7.11 mm, or 0.280 in. The baseline is at the same elevation as the base of the tree and the flying height above the base of the tree is 630 ft. From this information we wish to measure the height of tree G using each instrument.

Using the engineer's scale we get a tree-base separation of 2.35 in. and a treetop separation of 2.21 in., which gives us a difference in parallax, dP, of 0.14 in.

From our level-terrain height equation we calculate the height of the tree to be:

$$h = \frac{630 \text{ ft } (0.14 \text{ in.})}{0.28 \text{ in.} + 0.14 \text{ in.}} = 210 \text{ ft}$$

Now let's measure the same tree, using the parallax wedge, still without the sterescope. Using the parallax wedge without a stereoscope involves placing corres-

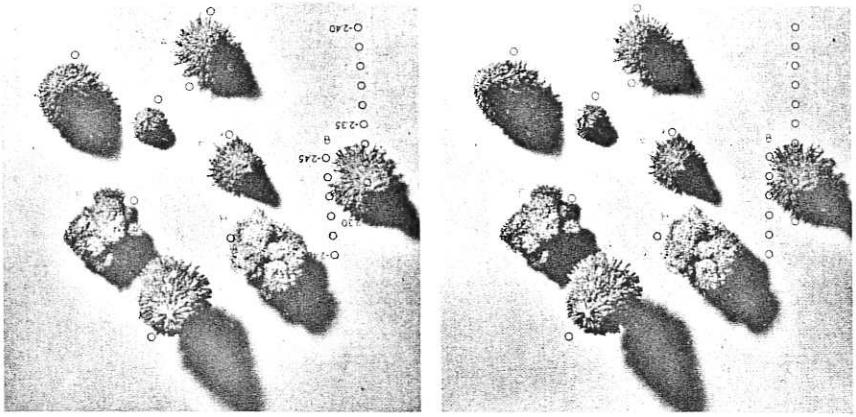


Figure 7.13. Stereogram of model trees for laboratory exercises. (Courtesy U.S. Forest Service, Pacific Northwest Forest and Range Experiment Station.)

ponding points on the wedge over images of the same point on each side of the stereogram. Doing this, we get a tree base separation of 2.34 in. and a treetop separation of 2.21 in., for a dP of 0.13. This time our tree height turns out to be 200 ft.

Now, once again, let's measure the same tree, but using the stereoscope, first with the parallax wedge and then with the parallax bar. Each of these methods should result in more precise answers. First, using the parallax wedge with the stereoscope, we get a tree-base separation of 2.346 in. and a treetop separation of 2.212 in. for a difference in parallax, dP, of 0.134 in. From our parallax equation we compute the height of the tree to be 204 ft. Now let's see what we get with the parallax bar.

Using the stereoscope, we turn the micrometer screw until the dot appears to float on the ground, then at the top of the tree. The results are that the tree base separation is 59.61 mm and the treetop separation is 56.18 mm for a dP of 3.43 mm. Because dP was measured in millimeters we must either convert to inches or we can leave dP in millimeters and simply use the previously given baseline length in millimeters of 7.11. Notice how the units cancel so that our final answer is in feet. Using this instrument the tree height is:

$$h = \frac{630 \text{ ft } (3.43 \text{ mm})}{7.11 \text{ mm} + 3.43 \text{ mm}} = 205 \text{ ft}$$

The last two methods are the most precise. The use of the engineer's scale and parallax wedge without the stereoscope gives us only approximate answers for the height of the tree. In fact in most situations a stereoscope with parallax wedge or bar is the only way heights can be satisfactorily measured on stereoscopic pairs. We used the letter G in Figure 7.13 for our ground measurement when not using the stereoscope. In actual practice we don't have any such letters although in rare instances we might use a stump, rock, or other discrete image on large-scale photographs. At most common scales of photography, the use of the parallax bar or wedge with a stereoscope is the only practical way to make height measurements except for approximate differences in elevation where these differences are large and where discrete conjugate images on both photos are clearly evident.

False Parallax

Another phenomenon that can influence the accuracy of height measurement and the appearance of the stereoscopic model is *false parallax*. This is caused by the slight movement of objects from one position to another while the plane is flying from one exposure station of a stereoscopic pair of photos to the next. An example is wind-sway of trees. The different position of tree tops on two successive photos will create an error in dP and therefore in the measured height. Foam, ice, or other debris floating down a river will sometimes cause the stereoscopic model to show the river as a curved surface because the debris in the center of the river moves a greater distance (between exposures) than the debris along the edges. Objects may move under their own power to create a false parallax as evidenced by the flying cows in Figure 7.16.

The following laboratory exercise will give you actual practice in making verical measurements on aerial photographs.

Laboratory Exercise

1. If tree F (Figure 7.13) is known to be 97 ft tall, calculate the height of tree A using the proportional shadow-length method.
2. Using the stereogram (Figure 7.13) make the necessary measurements and calculate the height of trees C and D. You may use an engineer's scale, parallax bar, parallax wedge, or all three instruments, but state which instrument or instruments you used. Assume H to be 630 ft, P to be 0.280 in. or 7.11 mm and that the level-terrain equation is appropriate.
3. Do exactly the same as question 2 except assume that ΔE is -50 ft (about 8 percent of H). Use the same measurements of dP.
4. Once again use the same measurements of dP but assume that ΔE is now *a* + 50 ft.
5. Obtain a stereoscopic pair of photos from your instructor of your area, determine the height of several objects and check these heights with field measurements.
6. Using the stereogram in Figure 7.14 determine (1) the road grade, and (2) the straight-line scope from points A to B. Use the level-terrain height equation. Additional information:

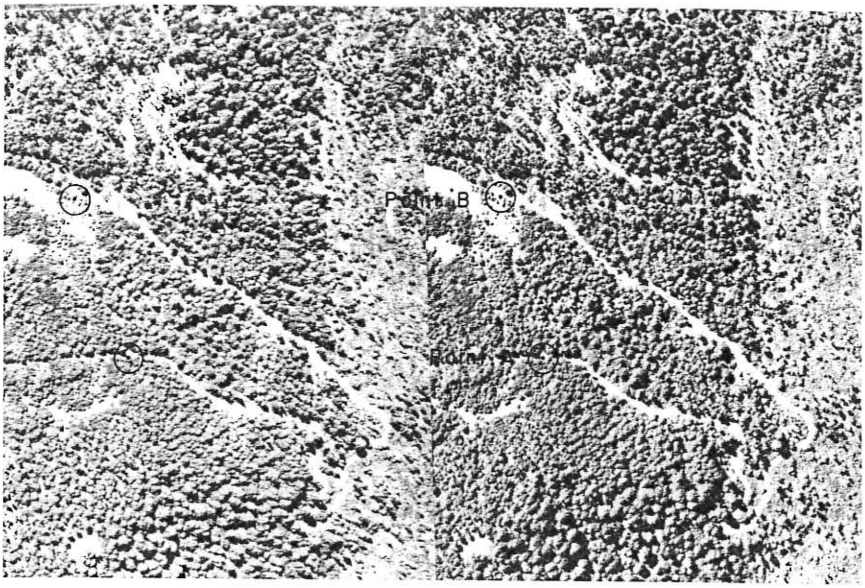


Figure 7.14 Stereogram for use with laboratory exercise 6. (From D.P. Paine, 1979, *An Introduction to Aerial Photography for Natural Resource Management,* O.S.U. Bookdstores, Inc., copyright 1973, reproduced with permission.

PSR at B = 12,050
Elevation at point B = 950 ft
Average photo baseline length (*P*) = 3.10 in.
Average elevation of baseline = 900 ft
Camera lens focal length = 12 in.

$$\text{Percent slope and percent grade} = 100\left(\frac{\text{vertical distance}}{\text{horizontal distance}}\right)$$

Hint: To get the correct ground horizontal distance you will have to first determine the average scale between points A and B. To do this you must first calculate the vertical distance between A and B.

Questions and Problems

1. The *absolute parallaxes* of points y and z are 3.28 in. and 3.12 in., respectively, and the average distance between the PP and CPP of the stereopair is 3.33 in. Which point, y or z, is the highest elevation? Why? What is the relative elevation of points y and z with respect to each other and to the average baseline elevation?

2. Given: A stereoscopic pair of photos and:

Average PSR of baseline	= 10,000
Average baseline length (P)	= 3.00 in.
Difference in absolute parallax between points A and B	= 0.50 in.
Average elevation of the principal points	= 2000 ft
Elevation of point A which is lower than B	= 1000 ft
Focal length of camera lens	= 6 in.

Calculate the elevation of point B using the most appropriate parallax equation.

3. In the solution to question 2 you should have calculated -0.50 in. for the second term in the denominator of the mountainous-terrain height equation. This is the same value as dP but with opposite sign. What should this immediately tell you about the elevation of point B without further calculation? Why?

4. Given: A stereoscopic pair of photos and:

Known height of a building	= 300 ft
Focal length of camera used	= 6 in.
Average distance between PP and CPP	= 2.87 in.
Difference in absolute parallax between the top and bottom of the building	= 2.92 mm
Elevation at the base of the building	= 250 ft
Average elevation of the two PPs	= 250 ft

Calculate the flying height of the aircraft above sea level (A) and the PSR at a point on the same photo with an elevation of 100 ft.

5. Given: A stereoscopic pair of photos and:

PSR	= 15,840
Focal length of camera lens	= 8¼ in.
Average distance between PPs and CPPs	= 84 mm
Parallax difference between the top and bottom of an object	= 0.92 mm

Determine the height of the object using Table 7.2 and the short-cut height equation if $\pm\Delta E$ is negligible.

6. On a stereoscopic pair of photos the dP of both trees A and B is measured to be 0.55 mm. On the ground, however, tree A is found to be 100 ft tall and tree B is only 90 ft. tall. Assuming the dP measurements to be correct, which tree is growing at the higher elevation? Why?

7. The distance on the *ground* between the principal and conjugate principal point for two perfectly vertical photographs of a stereoscopic pair are the same. Assume both photographs were taken at exactly the same flying height above sea level. The baseline (P) on the first photo is 3.05 in. and on the second photo it is 3.09 in. Which photo has the PP with the highest elevation? Why? Assume perfectly vertical photographs.

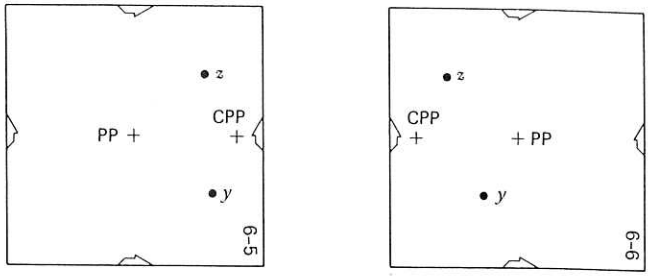


Figure 7.15. Stereogram for use with Problems 8 and 9.

8. What is the difference in absolute parallax between points y and z on the diagram in Figure 7.15? Which point is at the higher elevation? Why?
9. What is the difference in elevation between points y and z (Figure 7.15) if H is 10,000 ft? (Use an engineer's scale and the level terrain equation.)
10. Figure 7.16 shows some cows in a pasture. Because some of them moved between exposures, creating a false parallax difference, they appear to float above the ground. In which direction did they move (right or left)? Explain your answer. How would they appear in the stereogram if they had moved the opposite direction? Assume the plane flew from right to left.

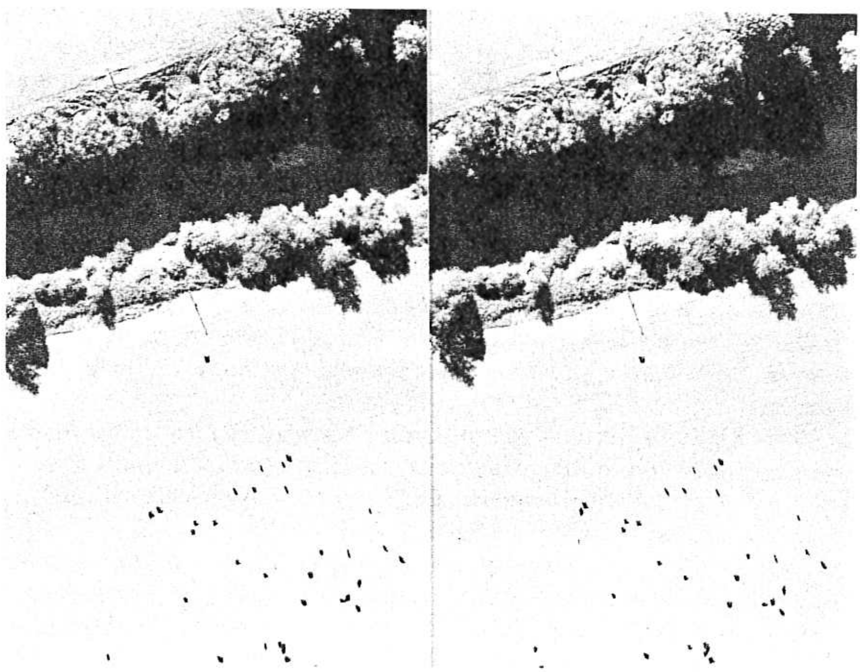


Figure 7.16. Birds or flying cows? See Problem 10 for answer.